Grade 6 Math Circles

October 4/5/6, 2022

Number Systems Problem Set

When converting between two positional number systems, neither of which is the decimal system, convert to decimal first and then to the other number system from decimal.

1. Evaluate the following powers.
   a) 5^3 b) 9^0 c) 8^2 d) 1^7 e) 16^1 f) 10^8

2. Convert the following to decimal (base 10).
   a) 5_8 b) D_{16} c) 1111_2 d) $A01F_{16}$ e) 10075_8 f) 1001110_2

3. Convert the following to binary (base 2).
   a) 111_{10} b) 7_8 c) $4E_{16}$

4. Convert the following to octal (base 8).
   a) 83_{10} b) 101011_2 c) $3F_{16}$

5. Convert the following to hexadecimal (base 16).
   a) 267_{10} b) 1111110_2 c) 2075_8

6. Determine whether the following pairs of numbers are equal.
   a) 10_{10} and 10_2 b) ABC_{16} and 2748_{10} c) 27_8 and 111_2 d) 1101110_2 and $6E_{16}$

7. How many symbols are used in a base n number system?

8. We can define a base 3 positional number system whose symbols are ◯, △, and □ using the following conversion chart.

Number System Symbols	◯	△	□
Decimal Values	0	1	2

The following exercises convert from the base 3 number system defined above to the decimal number system and vice versa. Recall that the methods used for both directions of conversion rely upon the base of the non-decimal number system, that is, the number of symbols.

a) Convert $\square \bigcirc \triangle\triangle\square_3$ to decimal.

b) Convert 104_{10} to the base 3 number system. Use the following conversion chart.

Base 10	$3^4 = 81$	$3^3 = 27$	$3^2 = 9$	$3^1 = 3$	$3^0 = 1$	Base 3

9. As mentioned in the lesson, languages and words are a lot like number systems and numbers. In fact, the letters of the alphabet can be considered as the symbols of a base-26 number system.

Letters	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*
Decimal Values	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Letters	*Q*	*R*	*S*	*T*	*U*	*V*	*W*	*X*	*Y*	*Z*
Decimal Values	16	17	18	19	20	21	22	23	24	25

Create your own symbols (at least 4) for a positional number system (do not use digits or letters from the Latin alphabet). Create a conversion chart from your number system to the decimal system. Then, write your initials as a number in your number system. (That is, convert from the base 26 system defined above to the decimal system, and from the decimal system to your own system.)

10. The simplest non-positional number system is the unary number system. The unary system contains one symbol, the digit 1. The positive integer n is represented by repeating the symbol n times. Since there is only one symbol, position has no effect on its value.

    a) What is the base of the unary number system?

    b) What is an example of a unary system that is commonly used?

    c) The unary system is very simple and only requires one symbol. What is a disadvantage of this number system?

    d) Does there exist a number system with a smaller base than the unary system?

11. A well-known non-positional number system is the Roman Number System. This system uses the following symbols from the Latin alphabet.

Roman Symbols	I	V	X	L	C	D	M
Decimal Values	1	5	10	50	100	500	1000

    The values of the numbers depend on the relative positions between the symbols.

    To convert from Roman numbers to decimal, start with the largest valued symbol and follow the two simplified rules below.

    - Smaller valued symbols are subtracted from larger ones when they are written to the left.
    - Smaller valued symbols are added to larger ones when they are written to the right.

    Convert the following into decimal.

    a) II b) IV c) VI d) XV e) IL f) CD g) $MMXXII$

12. Consider a base n positional number system which uses the digits, in their typical order, as its first ten symbols.

    a) Convert 1_n into the decimal system.

    b) Convert 10_n into the decimal system.

    c) Convert 100_n into the decimal system.

    d) Convert 1000_n into the decimal system.

    e) Convert $(10^e)_n$ into the decimal system, where e is any whole number.